$\boxed{262N1.0}$ P1 ~~2~~ ~~II 1~~

II Dimensional Aanalysis

i) Dimensional analysis handout. (Copies available on the course web site

ii) Estimating equm in the universe

Compare interaction rates with the expansion rate.

$$\Gamma = \underbrace{n}_{\text{number density of relevant particles}} \underbrace{\sigma}_{\text{cross-section}} |v|$$ = interaction (or reaction) rate

H = expansion rate

Case 1: Everything relativistic.

Only relevant energy scale in Γ is T using energy units, and dimensional analysis

$$\left.\begin{array}{l} n \sim T^3, \quad \sigma \sim T^{-2}, \\ |v| \approx 1 \end{array}\right\} \Gamma \sim T$$

$$H^2 = \frac{8\pi}{3} G\rho$$ (in early universe can neglect Λ and $\frac{K}{R_0^2 a^2}$)

$\rho \sim T^4$

write $G = m_p^{-2}$ ($m_p \approx 10^{19}$ GeV)

$H^2 \approx m_p^{-2} T^4$

When is $\Gamma >> H$?

$$T >> m_p^{-1} T^2$$

$$m_p >> T$$

so eqm achieved for $T << m_p \approx 10^{19}$ GeV

$\Rightarrow$ eqm possible starting in the very early universe.

Case 2: "Four Fermi" form for σ

There are many other possibilities than "Case 1", where everything was relativistic. Here is one example that illustrates an important point.

Some interaction are "mediated by a vector Boson" with mass M_x. In the limit where all interacting particles are relativistic but $T \ll M_x$, M_x appears with a "-4" power in σ.

to get the units right

$$\sigma = T^n M_x^{-4} \rightarrow n = 2$$

$$\Gamma \approx \frac{T^5}{M_x^4}$$

$$\Gamma \gg H \Rightarrow \frac{T^5}{M_x^4} \gg \frac{T^2}{m_p}$$

$$T \gg M_x \left(\frac{M_x}{m_p}\right)^{1/3}$$

for example, if $M_x \approx 100$ GeV ~~Tweak~~

(as it is for "weak interaction")

$T >> 10^{-4}$ GeV $= 100$ MeV

The point of choosing this particular "case 2" was to show that for some processes eqm is achieved for temeratures <u>above</u> some critical ~~val~~ value. The processes drop <u>out</u> of eqn as the expanding universe drops below the critical value.

~~iii) Properties of a freely propagating thermal distribution of photons.~~

~~Eq 2.28 from ~~Ryg~~ Ryden:~~

$$\cancel{\mathcal{E}(f)\,df = \frac{8\pi}{c^3}h\,\frac{f^3\,df}{\exp(hf/kT)-1}}$$

~~= ~~number~~ energy density of photons with frequencies between f and $f+df$.~~

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$m=0$

Particles in eqm:

Phase space distribution function

$$f(\vec{p}, t_1) = \left[\exp\left(E/T_1\right) \pm 1\right]^{-1}$$

$f(p) \cdot \cancel{\Delta} V \cdot d^3p$ = # of particles in vol dV with $\cancel{p}$ $p < p < p + dp$

~~Follow~~ For non-interacting particles:

follow vol dV and momentum between t_1 + t_2 (assume eqm at t_1)

For a photon traveling in an FRW universe
$p = \frac{1}{\lambda} \sim \frac{1}{a}$ (see p 40 in Ryden)

→ If we ~~follow~~ allow ΔV to evolve in a ~~comov~~ comoving way, # of particles is ~~the~~ unchanged, as long as we follow the momenta as they redshift:

$$\Delta V_1 \rightarrow \Delta V_2 = \Delta V_1 \left(\frac{a_2}{a_1}\right)^3$$

$$P_1 \rightarrow P_2 = P_1 \left(\frac{a_1}{a_2}\right)$$

$$f(\vec{p}_2\ t_2)\, dV_2\, d^3\vec{p}_2 = f(\vec{p}_1\ t_1)\, dV_1\, d^3\vec{p}_1$$

$$f(p_2\ t_2) = f(\vec{p}_1, t_1)\ \frac{dV_1}{dV_2}\ \frac{d^3\vec{p}_1}{d^3\vec{p}_2} = f(\vec{p}_1, t_1)$$

$$f(p_2\ t_2) = \left[\exp\left(p_1/T_1\right) \pm 1\right]^{-1} = \left[\exp\left(\frac{p_2\left(\frac{a_2}{a_1}\right)}{T_1}\right) \pm 1\right]^{-1}$$

$$T_2 \equiv T_1\left(\frac{a_1}{a_2}\right)$$

$$f(p_2\ t_2) = \left[\exp\left(p_2/T_2\right) \pm 1\right]^{-1}$$