Lecture 4c

Wednesday, January 8, 2020 8:49 PM

Continuing manifolds from Lecture 3b with further discussion

We cannot prove that gravity should be thought of as the curvature of spacetime; instead we can propose the idea, derive its consequences, and see if the result is a reasonable fit to our experience of the world. Let's set about doing just that.

(Carroll, p52)

To propose the idea, we need to introduce MANIFOLDS

- Carroll **section 2.1** contains a nice argument why the Equivalence Principle leads to curves space.
- MANIFOLDS
  - Extend the notion of space to allow for curvature and topology
  - Discuss with class intuitive notion of curvature based on life on the surface of the earth
    - Compare triangles "draped" onto the surface (sum of angles > 180deg) with "real triangles" which could cut through the surface.
    - A manifold can capture the notion of curvature *without* referencing an object (such as the surface of a sphere) in a larger (flat) space.
  - Discuss notions of topology with pictures from Carroll:

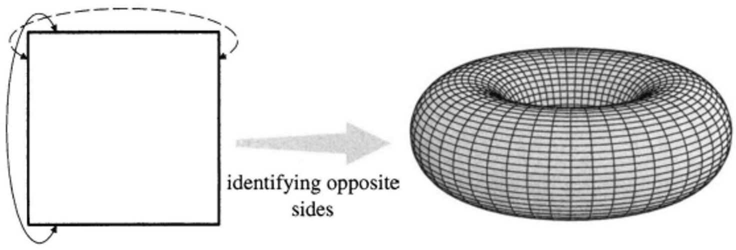


FIGURE 2.5 The torus, T^2, constructed by identifying opposite sides of a square.

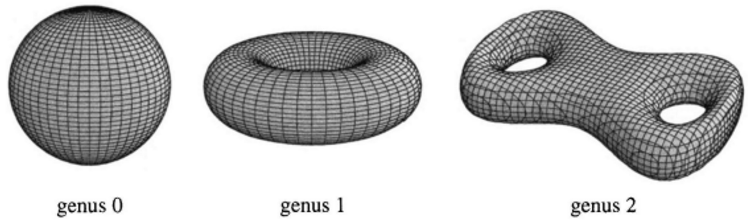
- 

  FIGURE 2.6 Riemann surfaces of different genera (plural of "genus").

- Manifolds need to have "nice" properties in order to do things like define functions, derivatives etc.
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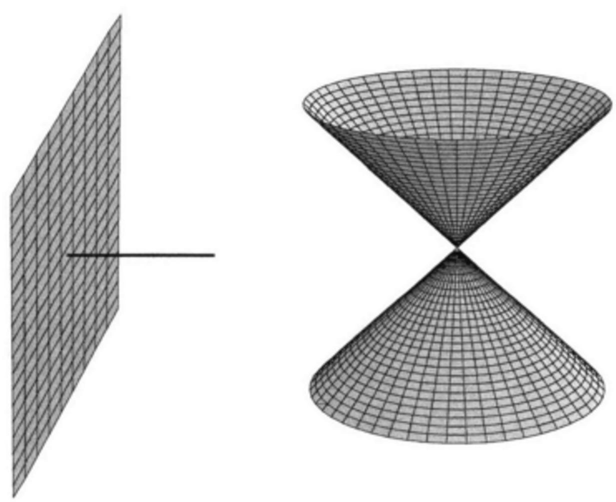
- LOCALLY EUCLIDEAN SPACE OF FIXED DIMENSION
  - Examples from the text:
  - 

    FIGURE 2.7 Examples of spaces that are not manifolds: a line ending on a plane, and two cones intersecting at their vertices. In each case there is a point that does not look locally like a Euclidean space of fixed dimension.

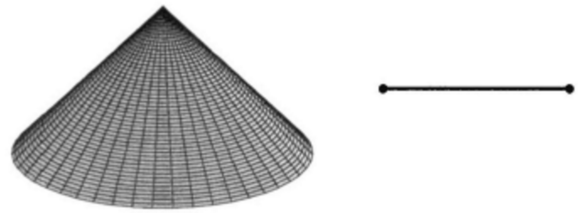
- 

  FIGURE 2.8 Subtle examples. The single cone is a smooth manifold, even though the curvature is singular at its vertex. A line segment is not a manifold, but may be described by the more general notion of "manifold with boundary."

  - Carroll says:
    - These subtle cases should convince you of the need for a rigorous definition, which we now begin to construct; our discussion follows that of Wald (1984).
  - I say lets nots not get into so much formal detail (skip the rest of section 2.2 starting at page 57

VECTORS ON A MANIFOLD (Carroll 2.3)

- Need to be careful about defining vectors LOCALLY, not stretching from here to there in the manifold
- The basic idea is to use "tangent vectors to curves" in the manifold.
- Giving this idea a coordinate invariant form leads to partial derivatives as the basis for the tangent space:
- Chapter 2 Manifolds

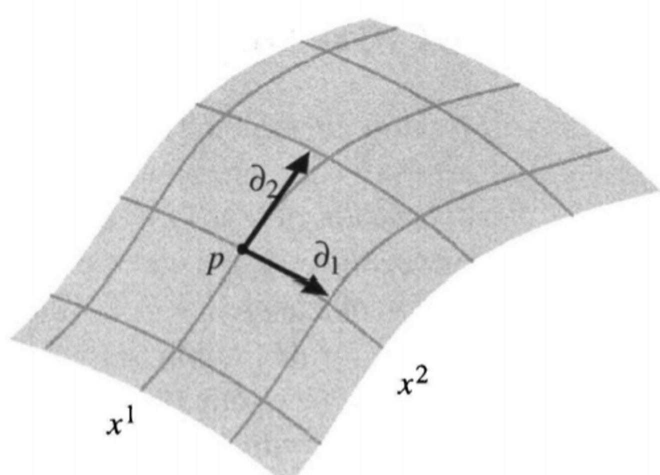
  

  FIGURE 2.18 Partial derivatives define directional derivatives along curves that keep all of the other coordinates constant.