

262W2.0

III

~~No Class Next week~~

III.1 Egm Thermo

number density:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3p$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3p$$

$$p = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E} f(\vec{p}) d^3p$$

~~Kinetic~~ $g =$ internal (e.g. spin) degrees of freedom

~~Kinetic~~ Egm:

$$f(\vec{p}) = \left[\exp((E - \mu)/T) \pm 1 \right]^{-1}$$

Fermi
Bose

262W2.0 (PI)

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~~No Class Next week~~

III.1 Eqm Thermo

number density:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3 p$$

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~~Kinet~~ $g =$ internal (e.g. spin) degrees of freedom

~~Kinet~~ Kinetic Eqm:

$$f(\vec{p}) = \left[\exp\left(\frac{E - \mu}{T}\right) \pm 1 \right]^{-1}$$

Fermi
Bose

E.G. Relativistic Limit: ($T \gg m$)

$$\rho = \left\{ \begin{array}{l} (\pi^2/30) g T^4 \quad (\text{Bose}) \\ (\pi^2/30) (7/8) g T^4 \quad (\text{Fermi}) \end{array} \right.$$

and $T \gg \mu$

$$\rho = \begin{array}{ll} \text{Bose} & \text{Fermi} \\ \pi^2/30 g T^4 & (7/8) \pi^2/30 g T^4 \end{array}$$

$$n = \pi^2 \zeta(3) \cancel{g} \cancel{T^2} g T^3 \quad 3/4 \times \dots$$

$$\rho = \rho/3$$

$$\zeta(3) = 1.202\dots \quad (\text{Riemann zeta fn.})$$

In general different species could be at different Temp (decoupled)

When ~~not~~ rad dom:

$$\rho = \frac{\pi^2}{30} g \star T^4$$

↑
photon temp

$$g_* \equiv \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermi}} g_i \left(\frac{T_i}{T}\right)^4$$

→ g_* can change as dist species go

non-relativistic

Go to other sheet.

III.2 Entropy

Assume eqm ⇒ "conservation of" entropy

~~entropy density: $s = \frac{S}{V} = \frac{\rho + P}{T}$~~

Entropy density = $\frac{\rho + P}{T}$ (see $k+T$)

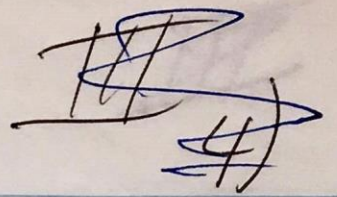
↳ Dominated by relativistic species:

$$s = \frac{2\pi^2}{45} g_* S T^3$$

$$g_* s \equiv \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermi}} g_i \left(\frac{T_i}{T}\right)^3$$

$$= T \rightarrow g_* s = g_*$$

262N2.0 (p4)



Important relations:

$$s = 1.80 g_{\text{xs}} n_{\gamma}$$

↖
photons

Today $s = 7.04 n_{\gamma}$ (std model)

but g_{xs} changes w/ time (rel \rightarrow nonrel)

K+T p 25

Conservation of s :

$$s \propto a^{-3}$$

$$= g_{\text{xs}} T^3 a^3 = \text{const}$$

when $g_{\text{xs}} = \text{const}$

$$a \Rightarrow T \sim \frac{1}{a}$$

Fig 3.5 (p65)

$$\text{or generally } T \propto g_{\text{xs}}^{-1/3} a^{-1}$$