262W2.0

III

1)

~~No Class Next week~~

III.1 Eqm Thermo

number density:

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p}) d^3p$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) f(\vec{p}) d^3p$$

$$P = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E} f(\vec{p}) d^3p$$

~~Kinet~~ g = internal (e.g. spin) degrees of freedom

Kinetic Eqm:

$$f(\vec{p}) = \left[\exp((E-\mu)/T) \pm 1\right]^{-1}$$

Fermi / Bose

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~~III~~

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(± : Fermi / Bose)

E.G. Relativistic Limit: $(T \gg m)$

~~$\rho = \begin{cases} (\pi^2/30)\, g\, T^4 & \text{(Bose)} \\ (7/8)(\pi^2/30 \end{cases}$~~

and $T \gg \mu$

	Bose	Fermi
$\rho =$	$\pi^2/30\, g T^4$	$(7/8)\, \pi^2/30\, g T^4$

$n = \pi^2 \zeta(3)\, g T^3 \quad 3/4 \times \ldots$

$p = \rho/3$

$\zeta(3) = 1.202\ldots$ (Riemann zeta fn.)

In general different species could be at different Temp (decoupled)

When rad dom:

$\rho = \frac{\pi^2}{30} g_* T^4$ ← photon temp

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~~III~~ ~~3)~~

$$g_* \equiv \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=\text{fermi}} g_i \left(\frac{T_i}{T}\right)^4$$

→ g_* can change as diff species go non-relativistic

Go to other sheet. →

III.2 Entropy

Assume eqm ⇒ "conservation of entropy"

~~entropy density: $S = \frac{S}{V} = \frac{\rho + P}{T}$~~

Entropy density $= \frac{\rho + P}{T}$ (see K+T)

Dominated by relativistic species:

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

$$g_{*s} \equiv \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 \frac{7}{8} \sum_{\text{Fermi}} g_i \left(\frac{T_i}{T}\right)^3$$

$= T \rightarrow g_{*s} = g_*$

Important relations:

$$s = 1.80\, g_{*s}\, n_\gamma$$

↑ photons

Today $s = 7.04\, n_\gamma$ (std model)

but g_{*s} changes w/ time (rel → nonrel)

~~K+T p75~~

Conservation of s:

$$s \propto a^{-3}$$

$$= g_{*s} T^3 a^3 = \text{const}$$

when $g_{*s} = \text{const}$ $\quad T \sim \frac{1}{a}$

Fig 3.5 (p65)

or generally $T \propto g_{*s}^{-1/3}\, a^{-1}$