Physics 262 Early Universe Cosmology
Homework 2
Assigned Wed April 7
Due Wed April 14 11pm
Corrections added 4/12 in blue

Background info: *As we discussed in class, one can divide the Friedmann equation by* ρ_c *to get a simple equation in terms of* Ω_i *'s. There is a related procedure that yields a similar expression that is different in very useful ways: Define* $\rho_c^{(100)}$ *as the value the critical density would have if* $H = 100 km/s/Mpc$. *Then define* $\omega_i \equiv \frac{\rho_i}{\rho_c^{(100)}}$. *The* ω_i *'s are technically just the* ρ_i *'s in a particular set of units. Note that* $\rho_c^{(100)}$ *is a constant, with no dependence on the value of the scale factor, Hubble constant, etc. Unlike* $\rho_{c,0}$, *the value of* $\rho_c^{(100)}$ *does not depend on any poorly measured quantities.*

2.1) Express the ω_i 's in terms of the Ω_i *'s* and the quantity $h \equiv \frac{H}{100 km/s/Mpc}$.

2.2) Rewrite the Friedmann equation so that it is only in terms of h and the ω_i 's

2.3) Find the value of $\rho_c^{(100)}$ in the units of density you used in Homework 1.

2.4) Write a modified version of the code you used in Homework 1 in terms of the ω_i 's (or in other words write the ρ_i 's in units of $\rho_c^{(100)}$). *There is not a separate item to be handed in for this problem specifically, but please use this version (with further modification for the subsequent problems) for the rest of this homework.*

2.5) Use the discussion of neutrino decoupling (for example on pages 74-76 of K&T) to correct your formula for ω_r in problem 2.4) to account for the presence of the neutrino background. Assume three species of massless neutrinos. Please write the corrected formula on your answer sheet and also modify your code to use this formula. *Hint, the discussion on pp 74-76 of K&T will be more understandable after a related discussion in the class lectures. If you work this problem before the associated lecture, it is ok to just skim those pages and extract the value of the energy density in photons plus neutrinos, which is given there.*

Comment: What I want you to do in 2.5 is calculate a better value for $\rho_{rad}(today)$ *by including the neutrinos. I still want you to then scale it as* a^{-4} *for other times. You will appreciate that this is not all that correct, since you are extrapolating to temperatures before the neutrino and photon temperatures became different. In fact, there are other problems with this simplification due to the fact* *that the number of relativistic species changes with temperature as mass thresholds are crossed (as we already discussed for the electrons). But I have decided not to ask you to model all of this for the homework.*

2.6) Repeat homework problem 1.3) but this time use the new formula for the radiation density you produced in problem 2.5).

2.7) Modify your code to include a non-zero value of ω_k and re-do the plots in problems 2.2 and 2.4 for the following two cases: $\omega_{k,0} = \pm 0.2\omega_{m,0}$. Add an additional curve to show the curvature density and produce separate plots for each of the two values of the curvature, for a total of four plots. Also, extend the x-axis into the future to *a=2* for all these plots. For the log-log plots you will need to plot the absolute value of the curvature density. *Important: Please do not change the values of $\rho_{i,0}$ you have already used above. Just add the curvature to what you have. That means the relationships between $\omega_{i,0}$ and the critical density given in HW2 will no longer hold precisely.*