Physics 262 Early Universe Cosmology

Homework 1

Assigned 3/31/21

Due 4/7/21 11pm (Submit files on Canvas)

1.1) Using formulas from the FRW handout, consider a model for the total density of they Universe given by

$$\rho(a) = \rho_r(a) + \rho_m(a) + \rho_\Lambda(a) \tag{1.1}$$

where subscript "r" refers to relativistic matter, "m" refers to non-relativistic matter and "Λ" refers to "cosmological constant matter".

Produce an expression for $\rho_i(a)$ for each of the above ρ_i's. Please use the convention that $a = 1$ today and use

$$\rho_m(1) = 0.28\rho_c(1) \tag{1.2}$$

$$\rho_\Lambda(1) = 0.72\rho_c(1) \tag{1.3}$$

and use $\rho_r(1)$ given by the density of a photon gas at $2.7K$. You may choose any units (correct for density) but please use the same units for all three! Use the value $H_0 = 65 km/s/Mpc$.

Hints:

1) Many of the numbers you need can be found in the appendices of K&T

2)There is a small inconsistency with the above set of assumption at the ~0.001% level which you might not even notice. But if you do, ignore it. We will come back to it later.

1.2) Plot $\log(\rho_i)$ vs $\log(a)$ for $i = r, m, \Lambda$. Please plot all three curves on the same plot. Take *a* in the interval $(10^{-6},1)$

1.3) Find the values a_{eq} for which

$$\rho_r(a_{eq}) = \rho_m(a_{eq}) \tag{1.4}$$

and a_Λ

$$\rho_m(a_\Lambda) = \rho_\Lambda(a_\Lambda) \tag{1.5}$$

1.4) Assuming the Universe is flat, Plot Ω_i vs $\log(a)$ for $i = r, m, \Lambda$ (all on the same plot, using the same range of *a*'s as in problem 1.2)). *Note: Please use a linear scale for the Ω axis.*

1.5) Integrate one of the equations from section 2 of the FRW notes to derive Eqn. 4.2 (from the same notes).