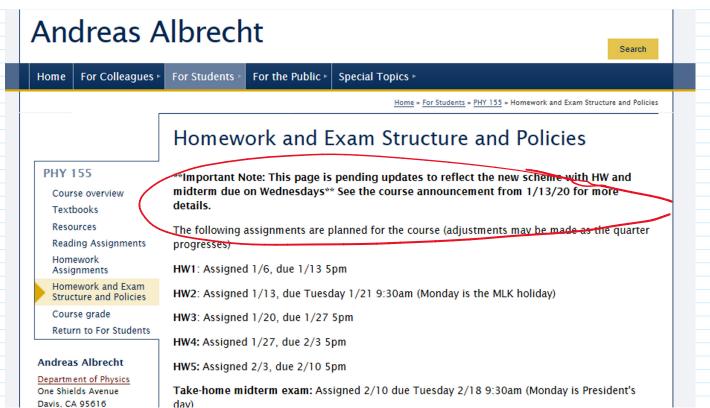
9:06 PM

Discuss: New Rules for HW due dates

Also: Midterm will also have a Wed due date



Reminder from Lecture 2:

- "A dual vector dotted with a vector is a scalar" (Less formal than Carroll)
 - This is a generalization of

A·B = Scalar (under 3d rotations).

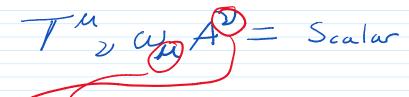
Which is simpler

because $R^{T} = R^{-1}$

TENSORS

Under 3D rotations:

Now let's extend the same idea to tensors (Also staying less formal than Carroll (section 1.6)):



Lower ==> dual vector, contracts with upper on tensor
Upper ==> vector, contracts with lower on tensor



ls a "type" (or "rank") (1,1) tensor

Is type (2,0)

Is type (0,2)

More formal stuff from Carroll:

tensor. Just as a dual vector is a linear map from vectors to \mathbf{R} , a tensor T of type (or rank) (k, l) is a multilinear map from a collection of dual vectors and vectors to \mathbf{R} :

$$T: T_p^* \times \cdots \times T_p^* \times T_p \times \cdots \times T_p \to \mathbf{R}.$$
 (1.56)

of ordered pairs of vectors Multilinearity means that the tensor acts linearly in each of its arguments; for instance, for a tensor of type (1, 1), we have

$$T(a\omega + b\eta, cV + dW) = acT(\omega, V) + adT(\omega, W) + bcT(\eta, V) + bdT(\eta, W).$$
(1.57)

space, we need to define a new operation known as the **tensor product** denoted by \otimes . If T is a (k, l) tensor and S is an (m, n) tensor, we define a (k + m, l + n) tensor $T \otimes S$ by

$$T \otimes S(\omega^{(1)}, \dots, \omega^{(k)}, \dots, \omega^{(k+m)}, V^{(1)}, \dots, V^{(l)}, \dots, V^{(l+n)})$$

$$= T(\omega^{(1)}, \dots, \omega^{(k)}, V^{(1)}, \dots, V^{(l)})$$

$$\times S(\omega^{(k+1)}, \dots, \omega^{(k+m)}, V^{(l+1)}, \dots, V^{(l+n)}). \tag{1.58}$$

As with vectors, we will usually take the shortcut of denoting the tensor T by its components $T^{\mu_1 \cdots \mu_k}_{\nu_1 \cdots \nu_l}$. The action of the tensors on a set of vectors and dual vectors follows the pattern established in (1.48):

$$T(\omega^{(1)},\ldots,\omega^{(k)},V^{(1)},\ldots,V^{(l)})=T^{\mu_1\cdots\mu_k}{}_{\nu_1\cdots\nu_l}\omega^{(1)}_{\mu_1}\cdots\omega^{(k)}_{\mu_k}V^{(1)\nu_1}\cdots V^{(l)\nu_l}.$$

(our To wu A")