$\boxed{262W3.0}$ (P1)

$\cancel{s = 2\pi^2}$

$sa^3 = \text{const}\ \cancel{= \frac{2\pi^2}{45} g_*}$

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

$$g_{*s} = \sum_{i=B} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{i=F} g_i \left(\frac{T_i}{T}\right)^3$$

$\left(\text{If } \forall i \ T_i = T \quad g_{*s} = g_* \right)$

$\cancel{Sa} = S = sa^3 = \text{const (eqm)}$

NB: s dominated by relativistic particles (only these included in $g_{*(s)}$)

$\sqrt[3]{s}\, a = \text{cost}$

$\cancel{\sqrt[3]{s}\, a =}$

$\sqrt[3]{g_{*s}}\, T a = \text{const}'$

$T \sim \frac{1}{a}$ when $g_{*s} = \text{const}$

Neutrino Decoupling:

$\bar{\nu}\nu \leftrightarrow e^+e^-$, $\nu e \leftrightarrow \nu e$ $\Rightarrow$ eqm

$$\sigma \simeq G_F^2 T^2, \quad n \simeq T^3$$

$$\Gamma_{int} = n\sigma|v| \simeq G_F^2 T^5$$

$$\frac{\Gamma_{int}}{H} \simeq \frac{G_F^2 T^5}{T^2/m_{pl}} \simeq \left(\frac{T}{1\,\text{MeV}}\right)$$

Decoupling at $T = 1$ MeV,) $T_\nu = T_\gamma$

Shortly below $T = 1$ MeV, $T < m_e$

$T \gg m_e$ $g_{*s} = T_\nu = T_\gamma = T_e$ $g_{*s} =$

$T \gtrsim m_e$ γ's: $g = 2$, $e^\pm$ ($g = 4$)

$$g_* = \frac{7}{8} \cdot 4 + 2 = \frac{11}{2} = g_{*s}$$

$T \ll m_e$ T_ν

for $T \gg 1$ MeV

Rel species = 3ν's $e^\pm$, γ's

$$g_* = \frac{7}{8}\cdot(3\cdot 2 + 2\cdot 2) + 2 = \frac{43}{4} \approx 11$$

$T \approx 1$ MeV Decoupling

Ingredients

$\boxed{1}$ ν's decoupled $\not{T}$

$\boxed{2}$ γ's coupled to rel. e's

$T_\nu = T_\gamma$

as the universe expands

$T_\nu \sim \frac{1}{a}$

electrons go non rel, remaining in eqm with γ's

conserve S ~~(γ-e system)~~

γ-e system has $\dot{S} = 0$

$g_{*s} T a^3 = \text{const}$

=

eqm

~~ν's~~ $e^{\mp}$'s γ's

$g_* = \frac{7}{8}(2 \cdot 2) + 2 = \frac{11}{2}$

Facts

i) g^{E}_{*s} includes rel particles only

ii) ~~all~~ contributions from rel. particles dominate S

later (e's non-rel)

$g^{E}_{*} = 2$

~~Before~~ t_1 e's rel : $g^{E}_{*}(t_1) = \frac{11}{2}$

t_2 e's non-rel $g^{E}_{*}(t_2) = 2$

$$\sqrt[3]{g^{E}_{*}(t_1)}\, T_1 a_1 = \sqrt[3]{g^{E}_{*}(t_2)}\, T_2^{\gamma} a_2$$

$$T_1^{\gamma} a_1 = \sqrt[3]{\frac{g^{E}_{*}(t_2)}{g^{E}_{*}(t_1)}}\, T_2^{\gamma} a_2$$

$$\sqrt[3]{\frac{4}{11}}$$

$$= T_2^{\nu} a_2 = \sqrt[3]{\frac{g^{E}_{*}(t_2)}{g^{E}_{*}(t_1)}}\, T_2^{\gamma} a_2$$

$$\frac{T_2^{\nu}}{T_2^{\gamma}} = \sqrt[3]{\frac{g^{E}_{*}(t_2)}{g^{E}_{*}(t_1)}} = \sqrt[3]{\frac{\frac{11}{2}}{2}} = \sqrt[3]{\frac{11}{4}}$$

late times

$$\rho = g_* \frac{\pi^2}{3} T^4$$

$$g_{*} = \frac{7}{8}\left[\underset{\substack{\uparrow \\ \text{species}}}{3}\cdot 2\left(\frac{T_\nu}{T_\gamma}\right)^4\right] + 2$$

$$= \frac{7}{8}\cdot 6 \cdot \left(\frac{11}{4}\right)^{4/3} + 2 \qquad = 3.36$$

$$g_{*s} = 2 + \left(\frac{7}{8} \times 2 \times 3\right)\left(\frac{T_\nu}{T_\gamma}\right)^3 = 3.91$$

N.B. Seprability of ν decoupling and e non-rel?