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II Dimensional Analysis

i) Dimensional analysis handout. (Copies available on the course web site

ii) Estimating equm in the universe

Compare interaction rates with the expansion rate.

$\Gamma = n \sigma |v|$ = interaction (or reaction) rate

n: number density of relevant particles

σ: cross-section

H = expansion rate

Case 1: Everything relativistic.

Only relevant energy scale in Γ is T using energy units, and dimensional analysis

$n \sim T^3$, $\sigma \sim T^{-2}$, $|v| \approx 1$ $\Big\}$ $\Gamma \sim T$

$H^2 = \frac{8\pi}{3} G \rho$ (in early universe can neglect Λ and $\frac{K}{R_0^2 a^2}$)

$\rho \sim T^4$

write $G = m_p^{-2}$ ($m_p \approx 10^{19}$ GeV)

$H^2 \approx m_p^{-2} T^4$

When is $\Gamma >> H$?

$T >> m_p^{-1} T^2$

$m_p >> T$

so eqm achieved for $T << M_p \approx 10^{19}$ GeV

$\Rightarrow$ eqm possible starting in the very early universe.

Case 2: "Four Fermi" form for σ

There are many other possibilities than "Case 1", where everything was relativistic. Here is ~~a~~ one example that illustrates an important ~~p~~ point.

Some interaction are "mediated by a vector Boson" with mass M_x. In the limit where all interacting particles are relativistic but $T \ll M_x$; M_x appears with a "-4" power in σ.

to get the units right

$$\sigma = T^n M_x^{-4} \rightarrow n = 2$$

$$\Gamma \simeq \frac{T^5}{M_x^4}$$

$$\Gamma \gg H \Rightarrow \frac{T^5}{M_x^4} \gg \frac{T^2}{m_p}$$

$$T \gg M_x \left(\frac{M_x}{M_p}\right)^{1/3}$$

for example, if $M_x \simeq 100$ GeV ~~[illegible]~~

(as it is for "weak interaction")

$T >> 10^{-4}$ GeV $= 100$ MeV

The point of choosing this particular "case 2" was to show that for some processes eqm is achieved for temperatures <u>above</u> some critical ~~[illegible]~~ value. The processes drop <u>out</u> of eqm as the expanding universe drops below the critical value.

iii) Properties of a freely propagating thermal distribution of photons.

Eq 2.25 from ~~Ryg~~ Ryden:

$$\mathcal{E}(f)\,df = \frac{8\pi}{c^3} h \frac{f^3\,df}{\exp(hf/kT) - 1}$$

= ~~number~~ energy density of photons with frequencies between f and $f + df$.

$m=0$

Particles in eqm:

Phase space distribution function

$$f(\vec{p}, t_1) = \left[\exp\left(E/T_1\right) \pm 1\right]^{-1}$$

$f(p) \cdot \cancel{\Delta}V \cdot d^3p$ = # of particles in vol dV with ~~p~~ $p < p < p + dp$

~~Follow~~ For non-interacting particles:

follow vol dV and momentum between t_1 & t_2 (assume eqm at t_1)

For a photon traveling in an FRW universe
$p = \frac{1}{\lambda} \sim \frac{1}{a}$ (see p 40 in Ryden)

→ If we ~~follow~~ allow ΔV to evolve in a ~~comoving~~ comoving way, # of particles is ~~the~~ unchanged, as long as we follow the momenta ~~& If~~ as they redshift:

$$\Delta V_1 \rightarrow \Delta V_2 = \Delta V_1 \left(\frac{a_2}{a_1}\right)^3$$

$$p_1 \rightarrow p_2 = p_1 \left(\frac{a_1}{a_2}\right)$$

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$$f(\vec{p}_2\, t_2)\, dV_2\, d^3\vec{p}_2 = f(\vec{p}_1\, t_1)\, dV_1\, d^3\vec{p}_1$$

$$f(p_2\, t_2) = f(\vec{p}_1, t_1)\, \frac{dV_1}{dV_c}\, \frac{d^3\vec{p}_1}{d^3\vec{p}_2} = f(\vec{p}_1, t_1)$$

$$f(p_2\, t_2) = \left[\exp\left(p_1/T_1\right) \pm 1\right]^{-1} = \left[\exp\left(\frac{p_2\left(\frac{a_2}{a_1}\right)}{T_1}\right) \pm 1\right]^{-1}$$

$$T_2 \equiv T_1\left(\frac{a_1}{a_2}\right)$$

$$f(p_2\, t_2) = \left[\exp\left(p_2/T_2\right) \pm 1\right]^{-1}$$