Physics 262 *Early Universe Cosmolog*y (A. Albrecht)
Homework 6
Assigned Feb 23
Due Mar 9

6.1) Consider your standard cosmological model from HW2/3. Evaluate the range of possible values of

$$\frac{\rho_k\left(T=10^{16}GeV\right)}{\rho_c\left(T=10^{16}GeV\right)} \tag{1.1}$$

that are consistent with $\left|\rho_k\right| < 0.1\rho_c$ today. For this purpose you may assume g_* is a constant throughout the history of the universe. *(Note, this last assumption is not a great approximation, but it does not interfere with the point of this problem.)*

6.2) Consider a comoving length that, when the temperature was $T = 10^{16} GeV$ has a physical length equal to the horizon distance at that time. What is the ratio of that comoving distance today to the Hubble length today? *Use the standard cosmology from HW2/3 and take g_* to be constant.*

6.3) Write a program to integrate K&T's equation 8.14. Assume ρ_φ (from eq 8.20, neglecting gradients) is the only contribution to H and use $V(\varphi) = \frac{1}{2}m^2\varphi^2$. Solve for $\log(a(t))$ (which is easier to solve for than $a(t)$) but allow the initial value $a(t_i)$ to be arbitrary. For $m = 10^{16} GeV$ (the grand unification scale) find a suitable initial value of φ and time range to see $\log(a(t))$ change by at least 100 during an inflationary period (when the energy density is potential dominated) and then follow at least two oscillations of φ following the end of the inflationary period. Make separate plots of $\varphi(t)$, $\log(a(t))$, $w(t)$, and

$$\left.\frac{\delta\rho}{\rho}\right|_H \equiv \delta_H = \left(\frac{HV'}{2\pi\dot{\varphi}^2}\right) \tag{1.2}$$

(from K&T Eqn 8.51) and $\rho_\varphi(t)$ for your solution. In your plot of $\rho(t)$ convert $\rho(t)$ to the same units you used for $\rho(a)$ calculated in earlier homeworks.

How do your values of δ_H during inflation ($w \approx -1$) compare with the realistic value $\delta_H \approx 10^{-5}$? (Note: Eqn (1.2) is only meaningful during inflation, not during the oscillating phase after inflation has ended)

Hints:

- *Give some thought to units. I found working with everything in GeV (including $G=m_p^{-2}$) was good, but you may have your own preference.*

- *Plot w(t) (use K&T equations 8.20 and 8.21 and recall that we are setting gradients to zero) and use trial and error to find a suitable initial conditions and time range*
- *You can choose* $\dot{\varphi}(t_i) = 0$.
- *You should be able to do this while keeping the integration time relatively short if needed. I chose* $t \in (0, 100m^{-1})$ *for fast turnaround, although your program probably can handle longer integration times without any trouble.*
- *Explore solution space by experimenting with initial values of* φ *rather than by lengthening the integration time.*
- *Use equations 8.20 and 8.21 to work out w(t) (remember we are neglecting gradients).*
- *The assumption that* ρ_φ *dominates H is justified after an initial period of inflation drives the universe to critical density (I don't want you to model that part however). .*

6.4) Repeat the procedure in problem 6.3 but this time vary *m* until you find a case where $\delta_H \approx 10^{-5}$ (within a factor of 3 or 4). Just show me the plots from your final solution.

6.5)

i) Inspect your solution in problem 6.4 and identify a "reheating time" t_r when *w* first starts to vary significantly from -1. Just make a choice from your plot. This does not need to be high-precision.

ii) Find $\rho_\varphi(t_r)$

iii) Consider the cosmological model from HW3. Suppose *g** remains constant for all $a < 10^{-6}$ (unrealistic, but good enough for our purposes). Find the value of the scale factor in this model where the total energy density equals $\rho_\varphi(t_r)$ from part ii). In what follows, we'll call this value a_r *Hint: You do not need to integrate for scale factors* $a < 10^{-6}$ *to answer this. Just use your solution from HW 2. and extrapolate in a simple way.*

iv) What is the temperature $T(a_r)$ in your model in part iii?

v) Consider the co-moving length scales with values 1Mpc, 100Mpc and 1000Mpc today. What values do these co-moving lengths when $a = a_r$? (Here I am basically asking you to report the values of the physical lengths at reheating by scaling, according to the rule for commoving lengths, back from the values I give for today.) What are the values of the scale factors when each of these co-moving lengths "re-enters" the Hubble length.

You now can consider the following simplified cosmological model that includes inflation: for $a > a_r$ the cosmology is given by your model in problem 6.5, part iii. For

$a < a_r$ the cosmology is given by your solution in problem 6.4. To get the scale factor right all you have to do is re-scale *a(t)* in your solution to 6.5iii) so that $a(t_r) = a_r$. This does not change the validity of the solution since the overall scale of *a* is just a matter of convention. Also, you need not worry about matching values of *t* at a_r. There is no need to define a global time coordinate. This cosmological model assumes instant "reheating": When $a = a_r$ the matter of the universe converts instantly from pure scalar field matter to the various matter components in your model form HW 3. (The reheating is not really "instant", but that assumption allows you to build a cosmological model that is pretty good for pedagogical purposes.)

6.6) Make a table of values of $\left.\frac{\delta\rho}{\rho}\right|_H$ for each of the co-moving lengths given in problem 6.5 part v (evaluated at the re-entry points determined in 6.5 part v). *Hint: To do this you will need to extrapolate back to the Hubble length exit time for each of these co-moving lengths. I expect this will be especially straightforward to work out graphically by plotting various relevant properties from above and finding Hubble exit by eye.*

6.7) Produce a table with three columns giving photon temperature in GeV, photon temperature in Kelvin, and time (in the time coordinate from the model form HW2) in units of seconds. The table should contain entries for the following events in your cosmological model:

- reheating,
- nucleosynthesis *(just use T=1MeV)*
- radiation-matter equality
- last scattering *(scale factor* $= \frac{1}{1100}$*)*
- the three Hubble length re-entry points you produced for problem 6.5 v).
- today

Hint: I think graphical methods similar to those suggested above might be useful here too.