Lecture 2a

Wednesday, January 8, 2020 8:49 PM

<u>News:</u> The library has acknowledged receipt of my reserve list for the class, and is processing it. I will keep you posted

<u>Reading</u>

- Section 2.1
- Section 2.2 up to (but not including) the paragraph that starts "these subtle cases" on p57
- Section 2.3 & 2.4

<u>Today: Vectors and Introduction to manifolds</u>

VECTORS:

Starting with material from Chapter 1 (Special relativity in Minkowski space) p17

Basis $\hat{e}_{(\mu)}$ parenthesis ⟹ a collection of four four-vectors

Write vector A as

$$A = A^{\mu}\hat{e}_{(\mu)}$$

Given a curve $x^{\mu}(\lambda)$

The tangent vector $V(\lambda)$ has components

$$V^{\mu} = \frac{dx^{\mu}}{d\lambda}$$

Under Lorentz transforms

$$x^{\mu} \to x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^{\mu}$$

$$= \Lambda^{\mu'}{}_{\mu} x$$

thus $V^{\mu} \to V^{\mu'} = \Lambda^{\mu'}{}_{\mu} V^{\mu}$

Learn how basis vectors transform by noting that the vector V is invariant:

$$V = V^{\mu} \underbrace{\hat{e}_{(\mu)}} = V^{\nu'} \hat{e}_{(\nu')} = \underbrace{\Lambda^{\nu'}{}_{\mu}} V^{\mu} \underbrace{\hat{e}_{(\nu')}}. \qquad (1.40)$$

$$\Rightarrow \hat{e}_{(\mu)} = \Lambda^{\nu'}{}_{\mu} \hat{e}_{(\nu')}$$

$\Rightarrow \hat{e}_{(\mu)}$ must transform as the <u>inverse</u> Lorentz transform.

Notation: Inverse of $\Lambda^{\mu'}{}_{\nu}$ written as $\Lambda^{\rho}{}_{\sigma'}$

(primes indicate the new coordinates and the primes switch places on the inverse)

DUAL VECTORS (Carroll section 1.5)

- If one wants to form an invariant (scalar) dot product, one of the vectors dotted needs to transform as the inverse Lorentz transform.
- Never worry about this with 3d rotations because

$$R^T = R^{-1} \text{ for these}$$

- Vectors that transform according to the inverse Lorentz transform are called "dual vectors" or "one-forms" written as

$$\omega_{\mu} \leftarrow \text{lower index}$$

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$$\omega_{\mu} \leftarrow \text{lower index}$$

- A basis for dual vectors has an upper index and is written:

$$\hat{\theta}^{(\nu)}$$

- With the property

$$\hat{\theta}^{(\nu)}\left(\hat{e}_{(\mu)}\right) = \delta^{\nu}_{\mu}$$

- A simple dual vector is a gradient of a scalar

$$d\phi = \underbrace{\frac{\partial \phi}{\partial x^{\mu}}}_{\text{The components of } d\phi} \hat{\theta}^{(\mu)}$$

Using the chain rule:

$$\frac{\partial \phi}{\partial x^{\mu'}} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial \phi}{\partial x^{\mu}}$$

$$\underbrace{= \Lambda^{\mu}{}_{\mu'} \frac{\partial \phi}{\partial x^{\mu}}}_{\text{confirming the dual vector transformation rule.}}$$