Physics 262 Early Universe Cosmology
Homework 5
Assigned Feb 7
Due Feb 22

These papers may be helpful:
https://arxiv.org/abs/astro-ph/9711102 (especially for problem 5.3)
https://arxiv.org/abs/astro-ph/9908085 (for problem 5.4)
However, it is possible to do fine without reading these papers.

5.1) For the following three cases, express the Friedmann eqn purely in terms of a, $\dot{a}$, and constants. Integrate to get an expression for $a(t)$. For the first two, use the convention $a(0)=0$. In each case, give your answer in terms of t_0 and a_0 $(=a(t_0))$.

i) A flat universe containing only Relativistic Matter
ii) A flat universe containing only Non-relativistic matter.
iii) A flat universe containing only ρ_Λ

5.2) The equation of sate for dark energy is often parameterized by the expression

$$w(a) = w_0 + w_a(1-a) \tag{1.1}$$

Derive an analytic expression for the dark energy density $\omega_Q(a)$ in terms of $\omega_{Q,0}$, w_0 and w_a.

5.3) Consider a homogeneous scalar field evolving according to K&T Eqn. (8.14), with $V(\varphi)=V_0 e^{-\lambda\varphi}$. *You also will need K&T Eqn (8.20) and Eqn (8,21) for what follows.*

a) Show analytically that if the only components of the Universe are non-relativistic matter and a homogeneous scalar field φ (and $\rho_k = 0$), a solution exists where ρ_φ remains a fixed fraction of ρ_m and $V(\varphi)=\frac{1}{2}\rho_\phi(\varphi)$. *Hint: You probably want to just do this by substitution.*

b) Give an expression for $\frac{\rho_\varphi}{\rho_{tot}}$ in terms of λ.

c) For what values of λ does your answer to b) make sense?

d) Verify that the "equation of state parameter" $\frac{p_\varphi}{\rho_\varphi}$ has the value it should for this solution.

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One model of dark energy has a homogeneous scalar field obeying K&T Eqn. (8.14) with

$$V(\varphi) = V_0\left(\chi\left(\varphi - \beta\right)^2 + \delta\right)e^{-\lambda\varphi} \tag{1.2}$$

The next few problems will deal with this case.

You should incorporate what we discuss about this model in class into your approach to problem 5.4.

5.4) Consider a simple two component model where made up of only ρ_m and ρ_φ, in the case where

$$\begin{aligned} \lambda &= 8 \\ \beta &= 34 \\ V_0 &= 1 \\ \delta &= 0.005 \\ \chi &= 1 \\ \rho_r &= \rho_k = 0 \end{aligned} \tag{1.3}$$

Here I use "reduced Planck units" where $8\pi G \equiv 1$. Solve K&T Eqn. (8.14) and experiment with a variety of initial values of φ. For each case I recommend that you choose an initial value for ρ_m that obeys the scaling solution you found in problem 5.3. This recommendation is just to offer you a starting point, and you will probably want to fiddle around with it to get a solution without too many transients. To hand in:

a) On the same graph, plot $V(\varphi)$ given by Eqn 1.2 in and $V(\varphi) = V_0 e^{-\lambda\varphi}$ for the parameters given in Eqn. (1.3). Chose a range for ϕ that includes $\varphi = \beta$ and extends far enough to either side of β that the two curves become similar (away from $\varphi = \beta$).

b) Find a numerical solution for $\varphi(t)$ for this potential with $\varphi \ll \beta$ that approximates your solution in 5.3a). Show explicitly with a plot or table that this is the case.

c) Find a solution where $w_\varphi(t) \to -1$ as time evolves, and plot the evolution of $w_\varphi(t)$ for this solution. *Hint: You will find this solution for values of φ not too far from β.*

d) Compare a value of ρ_φ from your solution in 5.4c where $w \approx -1$ with ρ_Λ from HW2.

e) Plot Ω_φ and Ω_m as a function of a or t (whatever is convenient) for the solutions in your answer to 5.4b) and 5.4c).

f) Make a single two panel plot showing the solutions you found in problem 5.4c. In the top panel plot φ on the x-axis and *t* or *a* on the y-axis. In the lower panel plot $V(\phi)$. Make sure the x-axis is the same on both panels. *This plot will help you see where the field is moving in the potential as a function of time*

<u>*Hints*</u>

i) *To do the numerical integration I recommend Matlab function "ode45".*

ii) *You will need to integrate simultaneously to get* $\rho_m(t)$*. One way to do this is to solve for* $a(t)$ *and use* $\rho_m \propto \frac{1}{a^3}$*. But it probably does not make sense to use the* $a_0 = 1$ *convention here. In particular, this homework is a theoretical exploration rather than a realistic model of the cosmos. I do not expect you to be concerned with which if any parts of your calculations might correspond to "today" (*$a = a_0$*).*