Continuing manifolds from Lecture 3b with further discussion

"We cannot prove that gravity should be thought of as the curvature of space-time; instead we can propose the idea, derive its consequences, and see if the result is a reasonable fit to our experience of the world. Let's set about doing just that."
(Carroll, p52)

To propose the idea, we need to introduce MANIFOLDS

- Carroll section 2.1 contains a nice argument why the Equivalence Principle leads to curves space.

- MANIFOLDS
  - Extend the notion of space to allow for curvature and topology
  - Discuss with class intuitive notion of curvature based on life on the surface of the earth
    - Compare triangles "draped" onto the surface (sum of angles > 180deg) with "real triangles" which could cut through the surface.
    - A manifold can capture the notion of curvature *without* referencing an object (such as the surface of a sphere) in a larger (flat) space.
  - Discuss notions of topology with pictures from Carroll:
Manifolds need to have "nice" properties in order to do things like define functions, derivatives etc.

**LOCALLY EUCLIDEAN SPACE OF FIXED DIMENSION**

- Examples from the text:
Carroll says:

These subtle cases should convince you of the need for a rigorous definition, which we now begin to construct; our discussion follows that of Wald (1984).

I say let’s not get into so much formal detail (skip the rest of section 2.2 starting at page 57).

VECTORS ON A MANIFOLD (Carroll 2.3)

- Need to be careful about defining vectors LOCALLY, not stretching from here to there in the manifold.
- The basic idea is to use "tangent vectors to curves" in the manifold.
- Giving this idea a coordinate invariant form leads to partial derivatives as the basis for the tangent space: