Physics 262 Early Universe Cosmology

Homework 1
Assigned 3/31/21
Due 4/7/21 11pm (Submit files on Canvas)

1.1) Using formulas from the FRW handout, consider a model for the total density of the Universe given by

\[ \rho(a) = \rho_r(a) + \rho_m(a) + \rho_\Lambda(a) \]  \hspace{1cm} (1.1)

where subscript “r” refers to relativistic matter, “m” refers to non-relativistic matter and “\( \Lambda \)” refers to “cosmological constant matter”.

Produce an expression for \( \rho_i(a) \) for each of the above \( \rho_i \)'s. Please use the convention that \( a = 1 \) today and use

\[ \rho_m(1) = 0.28 \rho_r(1) \] \hspace{1cm} (1.2)

\[ \rho_\Lambda(1) = 0.72 \rho_r(1) \] \hspace{1cm} (1.3)

and use \( \rho_r(1) \) given by the density of a photon gas at 2.7 K. You may choose any units (correct for density) but please use the same units for all three! Use the value \( H_0 = 65 \text{km/s/Mpc} \).

Hints:
1) Many of the numbers you need can be found in the appendices of K&T

2) There is a small inconsistency with the above set of assumption at the ~0.001% level which you might not even notice. But if you do, ignore it. We will come back to it later.

1.2) Plot \( \log(\rho_i) \) vs \( \log(a) \) for \( i = r, m, \Lambda \). Please plot all three curves on the same plot. Take \( a \) in the interval \((10^{-6}, 1)\)

1.3) Find the values \( a_{eq} \) for which

\[ \rho_r(a_{eq}) = \rho_m(a_{eq}) \] \hspace{1cm} (1.4)

and \( a_{\Lambda} \)

\[ \rho_m(a_{\Lambda}) = \rho_\Lambda(a_{\Lambda}) \] \hspace{1cm} (1.5)

1.4) Assuming the Universe is flat, Plot \( \Omega_i \) vs \( \log(a) \) for \( i = r, m, \Lambda \) (all on the same plot, using the same range of \( a \)'s as in problem 1.2)). \textit{Note: Please use a linear scale for the \( \Omega \) axis.}

1.5) Integrate one of the equations from section 2 of the FRW notes to derive Eqn. 4.2 (from the same notes).